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VELOCITY REGIMES OF MOTION OF A VISCOUS LIQUID IN A HORIZONTAL ROTATING CYLINDER

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The influence of viscosity on the stability of motion of a liquid tube in the cavity of a stationarily rotating horizontal cylinder is considered. An analytical condition of the stability of motion is obtained. The similarity criteria of steady motion of the liquid are selected. On the basis of experimental data diagrams of the transition boundaries for turbular and nontubular regimes of liquid motion are constructed.

Regimes of motion of a liquid in the cavity of a cylinder, rotating around the horizontal axis, substantially affect technological processes realized by rotating heat pipes [1, 2], drying steam drums, and centrifugal casting machines. An experimental analysis of the regimes of liquid motion with a low degree of filling of the cavity as applied to the hydromechanics of a condensate in drying steam cylinders of paper-making machines is given in [3, 4]. The problem of determining velocity regimes of tubular motion of an ideal liquid was analytically considered in [5]. The calculation for the parameters of motion of a viscous liquid tube with a small layer thickness as compared to the cylinder radius is numerically presented in [6, 2] on the basis of boundary layer theory. The work [7] describes characteristic regimes of liquid motion, taking account of disturbances, and attempts to generalize and extrapolate the results in the form of a two-parameter diagram.

The present work analytically deals with the influence of viscosity on the stability of tubular motion of a liquid. In this case the statement of the problem of [5] persists but the liquid is considered viscous. The possibility to find the similarity criteria and to obtain a universal diagram, determining the transition boundaries for the tubular and nontubular regimes, on the basis of experimental data is also dealt with.

We will consider a cylinder with radius R with smooth end walls, which uniformly rotates around the horizontal axis, perpendicular to the gravitational acceleration g, with the angular velocity ω and is partially filled with a liquid with the kinematic viscosity factor ν. Given a sufficient angular velocity of the cylinder, the liquid in the cavity takes the form of a tube of outer radius R and free surface radius cR $(0 \le c \le 1)$ (see Fig. 1).

The liquid motion is considered in a plane perpendicular to the axis of rotation of the cylinder. A polar system of coordinates r and φ is introduced; the velocity components are U and V. Then the equation of motion and the continuity conditions have the form

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial r} + \frac{V}{r}\frac{\partial U}{\partial \varphi} - \frac{V^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + g\cos\varphi +$$
$$+ \nu\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 U}{\partial \varphi^2} + \frac{1}{r}\frac{\partial U}{\partial r} - \frac{2}{r^2}\frac{\partial V}{\partial \varphi} - \frac{U}{r^2}\right);$$
$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial r} + \frac{V}{r}\frac{\partial V}{\partial \varphi} + \frac{UV}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial \varphi} - g\sin\varphi + \tag{1}$$
$$+ \nu\left(\frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{2}{r^2}\frac{\partial U}{\partial \varphi} - \frac{V}{r^2}\right);$$
$$\frac{1}{r}\frac{\partial}{\partial r}(rU) + \frac{1}{r}\frac{\partial V}{\partial \varphi} = 0,$$

where p is the pressure; ρ is the liquid density; t is the time. Surface tension is disregarded.

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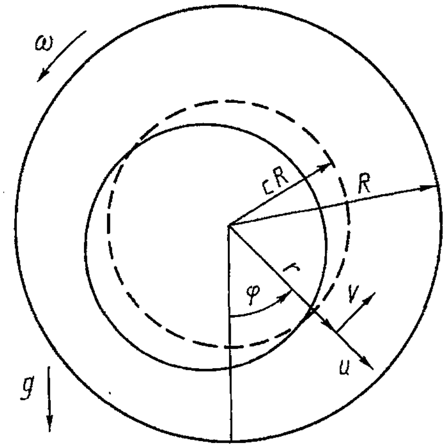

Fig. 1. Calculated scheme.

Uniform tubular solid rotation in the absence of gravitational forces is taken as undisturbed motion, velocities and pressure taking the values

$$U = 0, \ V = \omega r, \ p = \frac{1}{2} \rho \omega^2 (r^2 - c^2 R^2).$$

Gravitational forces trigger stationary disturbances of the velocity and pressure of the liquid in steady motion. Then, after the replacement of $\eta = r/R$ ($c \le \eta \le 1$) taking into account only normal stresses one can rearrange

$$\begin{gathered} U = \omega R U_0, \ V = \omega R (\eta + V_0), \\ p = \frac{1}{2} \rho \omega^2 R^2 [(\eta^2 - c^2) + p_0] + \rho g R \eta \cos \varphi, \end{gathered} \tag{2}$$

here U_0, V_0, and p_0 are the time-independent disturbances.

Let $\eta = c + \delta_0(\varphi)$ on the free surface, where δ_0 is the dimensionless stationary travel, small as compared to c.

With the disturbances U_0 and V_0 assumed small as compared to η, and $\partial^2 V/\partial r^2$ disregarded, the equations of disturbed motion based on (1) in view of (2) take the form

$$\begin{gathered} \frac{\partial U_0}{\partial \varphi} - 2V_0 = -\frac{1}{2} \frac{\partial p_0}{\partial \eta} + \frac{1}{\mathrm{Re}_0} \left(\eta^2 \frac{\partial^2 U_0}{\partial \eta^2} + \frac{\partial^2 U_0}{\partial \varphi^2} + \right. \\ \left. + \eta \frac{\partial U_0}{\partial \eta} - 2 \frac{\partial V_0}{\partial \varphi} - U_0 \right), \\ \frac{\partial V_0}{\partial \varphi} + 2U_0 = -\frac{1}{2\eta} \frac{\partial p_0}{\partial \varphi} + \frac{1}{\mathrm{Re}_0} \left(\frac{\partial^2 V_0}{\partial \varphi^2} + 2 \frac{\partial U_0}{\partial \varphi} \right), \\ \frac{\partial}{\partial \eta} (\eta U_0) + \frac{\partial V_0}{\partial \varphi} = 0. \end{gathered} \tag{3}$$

The boundary conditions respectively on the solid wall and the free surface become

$$\begin{gathered} U_0 = 0 \ \text{for} \ \eta = 1, \\ p_0 + 2c\delta_0 = -\frac{2c}{\mathrm{Fr}} \cos \varphi, \ U_0 = \frac{\partial \delta_0}{\partial \varphi} \ \text{for} \ \eta = c. \end{gathered} \tag{4}$$

After substituting

$$\delta_0 = \Delta \cos \varphi, \ p_0 = P(\eta) \cos \varphi, \ U_0 = \chi(\eta) \sin \varphi, \ V_0 = \xi(\eta) \cos \varphi \tag{5}$$

Eqs. (3) with the prescribed φ can be written as

$$\chi-2\xi=-\frac{1}{2}P'+\frac{\operatorname{tg}\varphi}{\mathrm{Re}_0}(\eta^2\chi''-2\chi+\eta\chi'+2\xi),$$
$$-\xi+2\chi=\frac{1}{2\eta}P+\frac{1}{\operatorname{tg}\varphi\,\mathrm{Re}_0}(2\chi-\xi), \tag{6}$$
$$\chi+\eta\chi'-\xi=0,$$

and the boundary conditions (4) as

$$\chi=0 \quad \text{for} \quad \eta=1,$$
$$P+2c\Delta=-\frac{2c}{\mathrm{Fr}},\quad \chi+\Delta=0 \quad \text{for} \quad \eta=c. \tag{7}$$

After rearranging (6) and eliminating P,

$$\eta^2\chi''+a\eta\chi'+b\chi=0, \tag{8}$$

where

$$a=\frac{3\,\mathrm{Re}_0\operatorname{tg}\varphi+3\operatorname{tg}^2\varphi-1}{\mathrm{Re}_0\operatorname{tg}\varphi+\operatorname{tg}^2\varphi-1};$$
$$b=\frac{1}{\mathrm{Re}_0\operatorname{tg}\varphi+\operatorname{tg}^2\varphi-1}.$$

The solution (8) has the form

$$\chi=A_1\eta^\alpha+A_2\eta^\beta, \tag{9}$$

here

$$\alpha=-d-\sqrt{d^2-b};\quad \beta=-d+\sqrt{d^2-b};$$
$$d=\frac{\mathrm{Re}_0\operatorname{tg}\varphi+\operatorname{tg}^2\varphi}{\mathrm{Re}_0\operatorname{tg}\varphi+\operatorname{tg}^2\varphi-1}.$$

From the first condition of (7) and (9) it follows that

$$A_1=-A_2=A$$

and then

$$\chi(\eta)=A(\eta^\alpha-\eta^\beta), \tag{10}$$

in view of (6)

$$\xi(\eta)=A[(\alpha+1)\eta^\alpha-(\beta+1)\eta^\beta],$$
$$P(\eta)=2A\eta[(\alpha-1)\eta^\alpha-(\beta-1)\eta^\beta]\left(\frac{1}{\mathrm{Re}_0\operatorname{tg}\varphi}-1\right). \tag{11}$$

In view of the third condition of (7) and (10)

$$\Delta=-A(c^\alpha-c^\beta) \tag{12}$$

and the second condition of (7)

$$A=-\frac{1}{\mathrm{Fr}}\left\{\frac{1}{[(\alpha-1)c^\alpha-(\beta-1)c^\beta]\left(\dfrac{1}{\mathrm{Re}_0\operatorname{tg}\varphi}-1\right)-(c^\alpha-c^\beta)}\right\}. \tag{13}$$

In the upper thickened portion of the tube gravitational and centrifugal forces are in opposition, and the necessary condition for the stability of stationarily disturbed motion of the tubular liquid layer is the positive value of the radial pressure gradient [5]. According to the numerical and experimental data of [6, 2] and our own experimental results, the largest thickening of the tube of a real viscous liquid occurs at the upper right of its cross section with $\pi/2 < \varphi < \pi$ (Fig. 1). Therefore on the basis of generalizing the known and obtained experimental data the dependence for the angle φ with the largest thickening of the tube wall, which corresponds to the minimum value of the radial pressure gradient on the free surface of the liquid, can be approximately represented as

$$\mathrm{tg}\,\varphi = -0{,}009c^{-1.5}, \tag{14}$$

with $\varphi = \mathrm{arctg}(-0.009\, c^{-1.5}) + \pi$.

From the third condition of (2), (11), and (13) it follows that

$$p = \frac{1}{2}\rho\omega^2 R^2 \left\{ (\eta^2 - c^2) - \frac{2\cos\varphi}{\mathrm{Fr}} \frac{[(\alpha-1)\eta^{\alpha+1} - (\beta-1)\eta^{\beta+1}]\left(\frac{1}{\mathrm{Re}_0\,\mathrm{tg}\,\varphi} - 1\right)}{[(\alpha-1)c^{\alpha} - (\beta-1)c^{\beta}]\left(\frac{1}{\mathrm{Re}_0\,\mathrm{tg}\,\varphi} - 1\right) - (c^{\alpha} - c^{\beta})} \right\} + \rho g R\eta\cos\varphi. \tag{15}$$

The necessary condition of stability in view of Eq. (15) has the form

$$\left(\frac{\partial p}{\partial \eta}\right) = \frac{1}{2}\rho\omega^2 R^2 \left\{ 2\eta - \frac{2\cos\varphi}{\mathrm{Fr}} \frac{[(\alpha-1)(\alpha+1)\eta^{\alpha} - (\beta-1)(\beta+1)\eta^{\beta}]\left(\frac{1}{\mathrm{Re}_0\,\mathrm{tg}\,\varphi} - 1\right)}{[(\alpha-1)c^{\alpha} - (\beta-1)c^{\beta}]\left(\frac{1}{\mathrm{Re}_0\,\mathrm{tg}\,\varphi} - 1\right) - (c^{\alpha} - c^{\beta})} \right\} + \rho g R\cos\varphi > 0 \quad \text{for } c + \delta_0 \leqslant \eta \leqslant 1. \tag{16}$$

This condition is realized on the free surface when in view of (5), (12), and (13)

$$\eta = c + \delta_0 = c + \frac{\cos\varphi}{\mathrm{Fr}} \left\{ \frac{(c^{\alpha} - c^{\beta})}{[(\alpha-1)c^{\alpha} - (\beta-1)c^{\beta}]\left(\frac{1}{\mathrm{Re}_0\,\mathrm{tg}\,\varphi} - 1\right) - (c^{\alpha} - c^{\beta})} \right\}. \tag{17}$$

Then on the basis of (16) and (17) after rearrangements the stability condition for the stationary motion of the liquid tube can be approximately represented as

$$\mathrm{Fr} > \frac{\cos\varphi}{c} \left\{ \frac{[(\alpha-1)\alpha c^{\alpha} - (\beta-1)\beta c^{\beta}]\left(\frac{1}{\mathrm{Re}_0\,\mathrm{tg}\,\varphi} - 1\right)}{[(\alpha-1)c^{\alpha} - (\beta-1)c^{\beta}]\left(\frac{1}{\mathrm{Re}_0\,\mathrm{tg}\,\varphi} - 1\right) - (c^{\alpha} - c^{\beta})} \right\}, \tag{18}$$

where $\mathrm{Re}_0 = \omega c^2 R^2/\nu$, and φ is determined from (14).

As $\nu \to \infty$ (an absolutely nonviscous liquid) $\mathrm{Re}_0 \to \infty$ and the condition (18), as was shown in [5], degenerates into

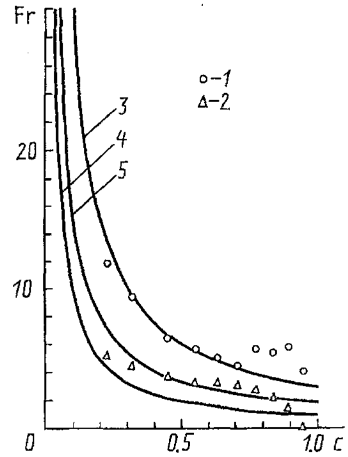

Fig. 2. Results of experimental determination of the stability conditions for motion of the liquid tube in comparison with calculated data: 1, 2) experimental data at R = 0.075 m respectively for $\nu = 10^{-6}$ m^2/sec (water) and $\nu = 10^{-3}$ m^2/sec (castor oil); 3) calculation from [5], and also from (18) as $\nu \to 0$ (Fr = 3/c); 4) calculation from (18) as $\nu \to \infty$ (Fr = 1/c); 5) calculation from (18) at $\nu = 10^{-3}$ m^2/sec and R = 0.075 m.

$$\mathrm{Fr} > \frac{3}{c}. \tag{19}$$

If $\nu \to \infty$ (an absolutely viscous liquid) $\mathrm{Re}_0 \to 0$, and the condition (18) becomes

$$\mathrm{Fr} > \frac{1}{c}. \tag{20}$$

Figure 2 shows the results of experimental determination of the stability conditions in comparison with the calculated data. In calculating the curves of limiting stability conditions from (18) in view of (14) the values of ω and Re_0 were determined by a successive approximation, and as a first approximation they were taken from the condition of zero viscosity of the liquid: Fr = 3/c. The limiting curve calculated from (18) at $\nu = 10^{-6}$ m^2/sec (water) and R = 0.075 m practically coincides with the line Fr = 3/c. The points on the plot correspond to the obtained experimental values of angular velocities, with which the destruction of the liquid tube during a slow decrease in the cylinder rotation velocity took place. Steady regimes above the curves correspond to a tubular form of motion, and those below - to a nontubular one.

The analysis of Fig. 2 shows good agreement of the results of solving Eq. (18) with the obtained experimental data.

Experimental investigations were performed on a plant fitted with nine renewable drums with the cavity radius range R = 0.01325-0.212 m. To visualize flow, one end wall of the drum was made transparent. Water and spindle and castor oils with the kinematic viscosity factor respectively 10^{-6}, $49 \cdot 10^{-6}$, and 10^{-3} m^2/sec served as working liquids. The degree of filling of the drum cavity with material κ (the material volume to cavity volume ratio) was realized in the form of ten discrete values and varied within $\kappa = 0.1$-0.95.

The angular velocities of the drum stationary rotation with formation and destruction of the liquid tube in steady motion were calculated, the first velocity being determined as the drum slowly accelerated from the quiescent state to the transition of the nontubular form of motion to the tubular one and the second - as the velocity of the drum with the liquid tube smoothly decreased to the transition of the tubular regime to the nontubular one. Nearly 500 points were obtained. Experimental data for $\kappa = 0.1$ are in good agreement with the results of [3, 4, 6].

Analysis of the experimental results has shown that in the given case the similarity criteria of steady motion of the liquid will be the Reynolds Re and Froude Fr numbers on the radial surface of the cylinder cavity as well as the degree of filling of the cavity with the liquid κ:

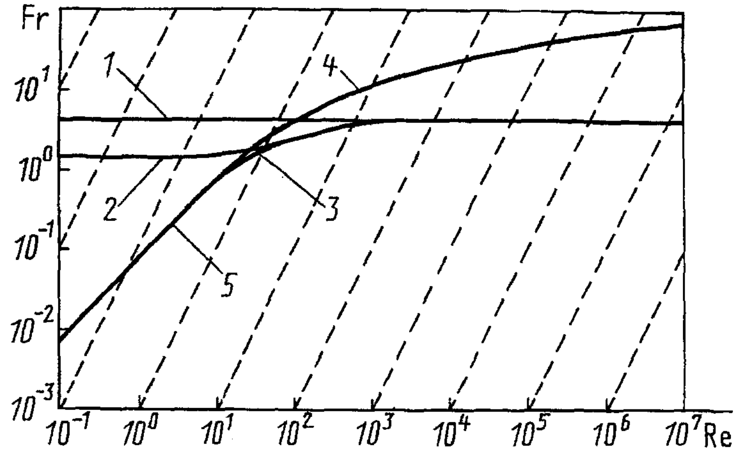

Fig. 3. Comparative diagram of the calculated and experimental determination of the transition boundaries for the tubular and nontubular regimes of liquid motion at $\kappa = 0.5$: 1, 2) calculation respectively from [5] and (18); 3, 4, 5) experimental data for the boundary of transition from, respectively, the tubular regime to the nontubular one with the cylinder run-out, the nontubular regime to the tubular one with acceleration, and the tubular regime to the nontubular one and back with run-out and acceleration.

$$\mathrm{Re} = \frac{\omega R^2}{\nu}, \quad \mathrm{Fr} = \frac{\omega^2 R}{g}, \quad \varkappa = \frac{\tau}{\pi R^2 L},$$

where τ is the volume of the liquid in the cylinder cavity; L is the cavity length, the first criterion characterizing frictional forces, the second - inertial forces, and the third - the geometric parameters of the motion.

Figure 3 presents a comparative diagrammatic analysis of the stability conditions of motion of the liquid tube from [5], the analytical results of this work, and the obtained experimental data. The transition boundaries for the tubular and nontubular regimes of liquid motion are plotted in the logarithmic axes Re and Fr for $\kappa = 0.5$. The portion of the diagram above the boundary corresponds to the tubular form of motion and that below - to the nontubular one. Inclined dashed lines correspond to the regimes of liquid motion in a cylinder of constant radius, rotating with different velocities. At large values of Re the phenomenon of hysteresis emerges - an excess of the cylinder rotation velocity in the formation of the tube during its acceleration over the destruction rate of the tube in its run-out [3, 4, 6, 2]. At small Re the rate of tube formation during acceleration and the rate of destruction during run-out are equal [6, 2], the indicated effects being due to the emergence of secondary circulation flows in the form of a roller on the inner surface of the tube.

The condition (18) obtained in the work in view of (14) corresponds closely to the experimental data for the angular rate of destruction of the tube in the run-out of the cylinder. However, this expression becomes incorrect as the secondary flows emerge.

Figure 4 gives universal diagrams of the transition boundaries for the regimes of liquid motion in the cylinder constructed on the basis of the obtained experimental data in the logarithmic axes Re and Fr for discrete values of κ: 0.1; 0.3; 0.5; 0.7, and 0.9. To determine with the aid of the diagrams the angular rotation velocity of the cylinder ω fitting the transition boundary of the regimes, from the two known parameters R and ν an inclined straight line, analogous to Fig. 3, is constructed for the current values of velocity. From the coordinates of the point of intersection of this straight line with the plot, for the corresponding degree of filling, ω is calculated. The angular velocity for the intermediate values of κ can be determined by interpolating.

Thus, an increase in the liquid viscosity decreases, according to (18) in view of (14), the value of the angular rotation velocity of the horizontal cylinder with which the tube loses stability and is destroyed and the liquid motion goes over into the nontubular form within the condition range from (19) to (20). The similarity criteria of steady motion are the Reynolds and Froude numbers on the cylinder surface as well as the degree of filling of the cavity. From the experimental data with allowance made for the similarity criteria one may construct universal diagrams of

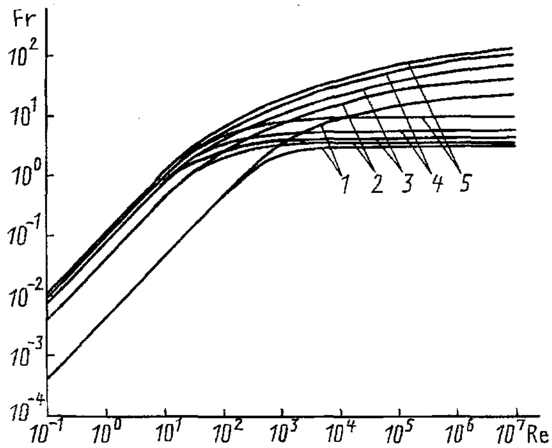

Fig. 4. Universal diagrams of the transition boundaries for the regimes of liquid motion: 1) $\kappa = 0.1$; 2) 0.3; 3) 0.5; 4) 0.7; 5) 0.9.

the transition boundaries for the regimes of liquid motion, which can be used to calculate velocity regimes of horizontal drum machines.

NOTATION

r, φ, polar coordinates; g, gravitational acceleration; ρ, density; ν, kinematic viscosity factor; R, radius of cylinder; c, ratio of the free surface of liquid tube in the cavity of cylinder to its radius; L, cylinder length; U, V, velocity components of liquid; ω, angular velocity of cylinder; p, pressure; U_0, V_0, disturbances of velocity; p_0, disturbance of pressure; δ_0, disturbance of the free surface of liquid tube; Δ, P, χ, ξ, variables in equations of disturbed motion; a, b, d, A_1, A_2, A, α, β, coefficients in equations of disturbed motion; $Re_0 = \omega c^2 R^2/\nu$, Reynolds number on the free surface of liquid tube; $Re = \omega R^2/\nu$, Reynolds number on the cylinder cavity surface; $Fr = \omega^2 R/g$, Froude number on the cylinder surface; $\kappa = \tau/\pi R^2 L$, degree of filling of cylinder with liquid.

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